

問 1 関数 $f(x, y) = 2x^2 + 2xy + y^2$ を考える. $z = f(x, y)$ のグラフ上の点 $(1, 1, f(1, 1))$ における接平面の方程式は

$$\boxed{1} x + \boxed{2} y - z = \boxed{3}$$

である.

$\boxed{1}$ ~ $\boxed{3}$ の解答群

- | | | | | | |
|-----|------|-----|------|-----|------|
| ① 1 | ② -1 | ③ 2 | ④ -2 | ⑤ 3 | ⑥ -3 |
| ⑦ 4 | ⑧ -4 | ⑨ 5 | ⑩ -5 | Ⓐ 6 | Ⓑ -6 |
| Ⓒ 7 | Ⓓ -7 | Ⓔ 8 | Ⓕ -8 | Ⓖ 9 | Ⓗ -9 |

計算用紙

問2 a, b を正の定数とする. xy 平面内の集合 D が

$$D = \left\{ (x, y) \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1, x \geq 0, y \geq 0 \right\}$$

で与えられているとき, 重積分

$$I = \iint_D xy \, dx dy$$

の値を求める. 変数変換 $x = ar \cos \theta, y = br \sin \theta$ を行くと, (r, θ) の集合

$$E = \left\{ (r, \theta) \mid 0 \leq r \leq \boxed{1}, 0 \leq \theta \leq \boxed{2} \right\}$$

は D に対応する. また, 変数変換のヤコビ行列式 (ヤコビアン) は

$$\begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \boxed{3}$$

であるので

$$I = \iint_E \boxed{4} \sin \theta \cos \theta \, dr d\theta = \boxed{5}$$

となる.

$\boxed{1} \cdot \boxed{2}$ の解答群

- | | | | | | |
|-------------------|-------------------|--------------------|---------|--------------------|----------|
| ⑤ $\frac{1}{4}$ | ① $\frac{1}{2}$ | ② $\frac{3}{4}$ | ③ 1 | ④ $\frac{3}{2}$ | ⑤ 2 |
| ⑥ $\frac{\pi}{4}$ | ⑦ $\frac{\pi}{2}$ | ⑧ $\frac{3\pi}{4}$ | ⑨ π | ⑩ $\frac{3\pi}{2}$ | ⑪ 2π |

$\boxed{3} \sim \boxed{5}$ の解答群

- | | | | | |
|------------|----------------------|----------------------|----------------------|------------------|
| ⑫ 0 | ⑬ ab | ⑭ $\frac{ab}{2}$ | ⑮ $\frac{ab}{4}$ | ⑯ $\frac{ab}{8}$ |
| ⑰ a^2b^2 | ⑱ $\frac{a^2b^2}{2}$ | ⑲ $\frac{a^2b^2}{4}$ | ⑳ $\frac{a^2b^2}{8}$ | ㉑ abr |
| ㉒ abr^2 | ㉓ abr^3 | ㉔ a^2b^2r | ㉕ $a^2b^2r^2$ | ㉖ $a^2b^2r^3$ |

計算用紙

問 3 3次元実ベクトル空間 \mathbb{R}^3 における 3つのベクトル

$$\mathbf{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{a}_2 = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}, \quad \mathbf{a}_3 = \begin{pmatrix} 5 \\ 7 \\ 9 \end{pmatrix}$$

が 1次独立であるかどうかを調べる.

そこで, c_1, c_2, c_3 を未知数とする方程式 $c_1\mathbf{a}_1 + c_2\mathbf{a}_2 + c_3\mathbf{a}_3 = \mathbf{0}$ を考えると,
連立 1次方程式

$$\begin{cases} c_1 + 3c_2 + 5c_3 = 0 \\ c_1 + 4c_2 + 7c_3 = 0 \\ c_1 + 5c_2 + 9c_3 = 0 \end{cases}$$

を得る. これを解くと,

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = t \begin{pmatrix} 1 \\ \boxed{1} \\ \boxed{2} \end{pmatrix} \quad (t \text{ は任意定数})$$

を得る. 特に $t = 1$ とおくと,

$$\mathbf{a}_1 + \boxed{1}\mathbf{a}_2 + \boxed{2}\mathbf{a}_3 = \mathbf{0}$$

が成り立つので, $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ は $\boxed{3}$.

$\boxed{1}$ ・ $\boxed{2}$ の解答群

- ① 0 ② 1 ③ 2 ④ 3 ⑤ 4 ⑥ 5
⑦ -1 ⑧ -2 ⑨ -3 ⑩ -4 ⑪ -5

$\boxed{3}$ の解答群

- ① 1次独立である ② 1次従属である
③ 1次独立であるとも 1次従属であるとも言えない

計算用紙

問 4 行列 $A = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$ の対角化について考える.

- (1) A の固有値を λ_1, λ_2 ($\lambda_1 < \lambda_2$) とすると, $(\lambda_1, \lambda_2) = \boxed{1}$ である. また, λ_1 に対応する固有ベクトルとして $\mathbf{u} = \begin{pmatrix} -1 \\ \boxed{2} \end{pmatrix}$, λ_2 に対応する固有ベクトルとして $\mathbf{v} = \begin{pmatrix} 1 \\ \boxed{3} \end{pmatrix}$ がとれる.

- (2) (1) で求めた固有ベクトル \mathbf{u}, \mathbf{v} の \mathbf{u} を第 1 列, \mathbf{v} を第 2 列にもつ行列を

$$P = (\mathbf{u} \ \mathbf{v}) = \begin{pmatrix} -1 & 1 \\ \boxed{2} & \boxed{3} \end{pmatrix}$$

とすると, $P^{-1}AP$ は対角行列となり, その (2, 2) 成分は $\boxed{4}$ となる.

$\boxed{1}$ の解答群

- | | | | |
|-----------|-----------|-----------|-----------|
| ① (0, 1) | ② (1, 2) | ③ (1, 3) | ④ (2, 3) |
| ⑤ (-1, 1) | ⑥ (-1, 2) | ⑦ (-1, 3) | ⑧ (-2, 3) |

$\boxed{2} \sim \boxed{4}$ の解答群

- | | | | | |
|------|------|------|------|------|
| ① 0 | | | | |
| ② 1 | ③ 2 | ④ 3 | ⑤ 4 | ⑥ 5 |
| ⑦ -1 | ⑧ -2 | ⑨ -3 | ⑩ -4 | ⑪ -5 |

計算用紙

(注意) y は x の関数であり, y', y'' は y の導関数 $\frac{dy}{dx}, \frac{d^2y}{dx^2}$ を表す. また, 特殊解は特解ともいう.

問 5 微分方程式

$$(*) \quad y'' - 2y' + 2y = 2x^2$$

の解 $y(x)$ で, 初期条件

$$(**) \quad y(0) = 0, \quad y'(0) = 0$$

を満たすものについて考える.

(1) $(*)$ に対応する同次方程式

$$y'' - 2y' + 2y = 0$$

の一般解 y_h は任意定数 C_1, C_2 を用いて

$$y_h = \boxed{1}$$

と表せる.

1 の解答群

- | | |
|----------------------------------|-------------------------------------|
| ① $C_1 \cos x + C_2 \sin x$ | ⑧ $x(C_1 \cos x + C_2 \sin x)$ |
| ② $e^x(C_1 \cos x + C_2 \sin x)$ | ⑨ $e^{-x}(C_1 \cos x + C_2 \sin x)$ |
| ③ $C_1 + C_2 e^x$ | ⑩ $C_1 + C_2 e^{-x}$ |
| ④ $(C_1 + C_2 x)e^x$ | ⑪ $(C_1 + C_2 x)e^{-x}$ |

(2) $(*)$ の特殊解で, 定数 a, b, c を用いて

$$y_p = ax^2 + bx + c$$

と表されるものを求めると, $a = \boxed{2}$, $b = \boxed{3}$, $c = \boxed{4}$ となる.

(3) (*) の一般解は

$$y = y_h + y_p$$

であるから、初期条件 (**) を満たすように C_1, C_2 を定めると

$$C_1 = \boxed{5}, \quad C_2 = \boxed{6}$$

となる.

2 ~ **6** の解答群

- | | | | | |
|-----|------------------|------------------|------------------|------------------|
| ① 0 | ② 1 | ③ 2 | ④ 3 | ⑤ 4 |
| | ⑥ -1 | ⑦ -2 | ⑧ -3 | ⑨ -4 |
| | ⑩ $\frac{1}{2}$ | Ⓐ $\frac{3}{2}$ | Ⓑ $\frac{5}{2}$ | Ⓒ $\frac{7}{2}$ |
| | Ⓓ $-\frac{1}{2}$ | Ⓔ $-\frac{3}{2}$ | Ⓕ $-\frac{5}{2}$ | Ⓖ $-\frac{7}{2}$ |

計算用紙

Applied Mathematics Examination Booklet

Entrance Examination for Master Course of Engineering, Graduate School of Sciences and Technology for Innovation, Yamaguchi University

2nd Enrollment Session for April 2026

Regarding the examination booklet

1. After starting the examination, fill out your examinee's number in the designated field on this booklet.
2. At the end of the examination, this booklet will be collected.

Regarding problem selection

1. Problems 1 to 4 are compulsory.
2. Problems 5 and 6 are elective. Select one and solve it along with the four compulsory problems.
3. Write the problem number of the elective problem you selected in the designated field on your answer sheet.

Regarding problem notation

1. Numbers enclosed in dashed lines represent numbers that have appeared previously. For example, the answer for $\boxed{\text{3}}$ is the same as for $\boxed{\text{3}}$.
2. For mathematical expressions, $\boxed{\text{3}}$ equals $(\boxed{\text{3}})$. For example, if $\boxed{\text{3}}$ equals $-x - 1$, then $x^2 - \boxed{\text{3}}$ equals $x^2 - (-x - 1)$.
3. \mathbb{R} represents the whole set of real numbers.
4. $\log x$ is the natural logarithm of x , namely, the logarithm $\log_e x$ on base e .

Regarding the answer sheet

1. Select the most appropriate answer from the specified answer choices, and write the answer symbol (for example ① ②) in the designated field on your answer sheet. However, if there is no appropriate answer, write ③ instead.

Examinee's No.							
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Problem 1 Consider the function

$$f(x, y) = 2x^2 + 2xy + y^2.$$

The equation of the tangent plane to the graph of $z = f(x, y)$ at the point $(1, 1, f(1, 1))$ is

$$\boxed{\mathbf{1}} x + \boxed{\mathbf{2}} y - z = \boxed{\mathbf{3}} .$$

Answer Choices $\boxed{\mathbf{1}} \sim \boxed{\mathbf{3}}$

- | | | | | | |
|-----|------|-----|------|-----|------|
| Ⓐ 1 | Ⓐ -1 | Ⓐ 2 | Ⓐ -2 | Ⓐ 3 | Ⓐ -3 |
| Ⓑ 4 | Ⓑ -4 | Ⓑ 5 | Ⓑ -5 | Ⓑ a | Ⓑ b |
| Ⓒ 7 | Ⓒ -7 | Ⓒ 8 | Ⓒ f | Ⓒ 9 | Ⓒ h |

This page may be used for calculations.

Problem 2 Let a and b be positive constants. We evaluate the value of the double integral

$$I = \iint_D xy \, dx \, dy$$

over the region D in the xy -plane given by

$$D = \left\{ (x, y) \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1, x \geq 0, y \geq 0 \right\}.$$

By making change of variables $x = ar \cos \theta$ and $y = br \sin \theta$, the region E in terms of (r, θ) ,

$$E = \left\{ (r, \theta) \mid 0 \leq r \leq \boxed{1}, 0 \leq \theta \leq \boxed{2} \right\},$$

corresponds to D . Furthermore, the Jacobian determinant of the transformation is

$$\begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \boxed{3}.$$

Hence it follows that

$$I = \iint_E \boxed{4} \sin \theta \cos \theta \, dr \, d\theta = \boxed{5}.$$

Answer Choices $\boxed{1}$ • $\boxed{2}$

- | | | | | | |
|-------------------|-------------------|--------------------|---------|--------------------|----------|
| Ⓐ $\frac{1}{4}$ | Ⓐ $\frac{1}{2}$ | Ⓐ $\frac{3}{4}$ | Ⓐ 1 | Ⓐ $\frac{3}{2}$ | Ⓐ 2 |
| Ⓑ $\frac{\pi}{4}$ | Ⓑ $\frac{\pi}{2}$ | Ⓑ $\frac{3\pi}{4}$ | Ⓑ π | Ⓑ $\frac{3\pi}{2}$ | Ⓑ 2π |

Answer Choices 3 ~ 5

- | | | | | |
|------------|----------------------|----------------------|----------------------|------------------|
| ① 0 | ② ab | ③ $\frac{ab}{2}$ | ④ $\frac{ab}{4}$ | ⑤ $\frac{ab}{8}$ |
| ⑥ a^2b^2 | ⑦ $\frac{a^2b^2}{2}$ | ⑧ $\frac{a^2b^2}{4}$ | ⑨ $\frac{a^2b^2}{8}$ | ⑩ abr |
| ⑪ abr^2 | ⑫ abr^3 | ⑬ a^2b^2r | ⑭ $a^2b^2r^2$ | ⑮ $a^2b^2r^3$ |

Problem 3 We investigate whether the following three column vectors on the 3-dimensional real vector space \mathbb{R}^3 ,

$$\mathbf{a}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{a}_2 = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}, \quad \mathbf{a}_3 = \begin{pmatrix} 5 \\ 7 \\ 9 \end{pmatrix},$$

are linearly independent. To do so, we consider the equation

$$c_1 \mathbf{a}_1 + c_2 \mathbf{a}_2 + c_3 \mathbf{a}_3 = \mathbf{0}$$

with unknowns c_1, c_2, c_3 . This leads to the system of linear equations

$$\begin{cases} c_1 + 3c_2 + 5c_3 = 0, \\ c_1 + 4c_2 + 7c_3 = 0, \\ c_1 + 5c_2 + 9c_3 = 0. \end{cases}$$

Solving this system, we obtain

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = t \begin{pmatrix} 1 \\ \boxed{1} \\ \boxed{2} \end{pmatrix} \quad (t \text{ is an arbitrary constant}).$$

In particular, by setting $t = 1$, we find that

$$\mathbf{a}_1 + \boxed{1} \mathbf{a}_2 + \boxed{2} \mathbf{a}_3 = \mathbf{0}$$

holds. Hence the vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ are $\boxed{3}$.

Answer Choices $\boxed{1} \cdot \boxed{2}$

- ① 0 ② 1 ③ 2 ④ 3 ⑤ 4 ⑥ 5
 ⑦ -1 ⑧ -2 ⑨ -3 ⑩ -4 ⑪ -5

Answer Choices 3

- ① linearly independent ① linearly dependent
- ② neither linearly independent nor linearly dependent

Problem 4 Consider the diagonalization of matrix $A = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$.

- (1) Let λ_1 and λ_2 be the eigenvalues of A with $\lambda_1 < \lambda_2$. Then we have $(\lambda_1, \lambda_2) = \boxed{1}$.

In this case, we can take an eigenvector $\mathbf{u} = \begin{pmatrix} -1 \\ \boxed{2} \end{pmatrix}$ corresponding to λ_1 , and an eigenvector $\mathbf{v} = \begin{pmatrix} 1 \\ \boxed{3} \end{pmatrix}$ corresponding to λ_2 .

- (2) Let P be the matrix whose first and second columns are the eigenvectors \mathbf{u} and \mathbf{v} obtained in (1), respectively. Namely, P is given by

$$P = (\mathbf{u} \ \mathbf{v}) = \begin{pmatrix} -1 & 1 \\ \boxed{2} & \boxed{3} \end{pmatrix}.$$

Then $P^{-1}AP$ is a diagonal matrix, and its (2, 2)-entry is $\boxed{4}$.

Answer Choices $\boxed{1}$

- Ⓐ (0, 1) Ⓐ (1, 2) Ⓐ (1, 3) Ⓐ (2, 3)
 Ⓐ (-1, 1) Ⓐ (-1, 2) Ⓐ (-1, 3) Ⓐ (-2, 3)

Answer Choices $\boxed{2} \sim \boxed{4}$

- Ⓐ 0
 Ⓐ 1 Ⓐ 2 Ⓐ 3 Ⓐ 4 Ⓐ 5
 Ⓐ -1 Ⓐ -2 Ⓐ -3 Ⓐ -4 Ⓐ -5

This page may be used for calculations.

Note. y is a function of x . The derivatives $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ are denoted by y' and y'' , respectively.

Problem 5 Consider the solution $y(x)$ of the differential equation

$$(*) \quad y'' - 2y' + 2y = 2x^2$$

with the initial conditions

$$(**) \quad y(0) = 0 \quad \text{and} \quad y'(0) = 0.$$

(1) The general solution y_h of the homogeneous equation corresponding to (*), namely

$$y'' - 2y' + 2y = 0,$$

is given by

$$y_h = \boxed{1}$$

with arbitrary constants C_1 and C_2 .

Answer Choices

- | | |
|----------------------------------|-------------------------------------|
| ① $C_1 \cos x + C_2 \sin x$ | ① $x(C_1 \cos x + C_2 \sin x)$ |
| ② $e^x(C_1 \cos x + C_2 \sin x)$ | ③ $e^{-x}(C_1 \cos x + C_2 \sin x)$ |
| ④ $C_1 + C_2 e^x$ | ⑤ $C_1 + C_2 e^{-x}$ |
| ⑥ $(C_1 + C_2 x)e^x$ | ⑦ $(C_1 + C_2 x)e^{-x}$ |

(2) We find a particular solution of (*) that can be written as

$$y_p = ax^2 + bx + c$$

with constants a, b, c . This leads to $a = \boxed{2}$, $b = \boxed{3}$, $c = \boxed{4}$.

(3) The general solution of (*) is

$$y = y_h + y_p.$$

Therefore, the constants C_1 and C_2 that satisfy the initial conditions (**) are

$$C_1 = \boxed{5} \quad \text{and} \quad C_2 = \boxed{6}.$$

Answer Choices $\boxed{2} \sim \boxed{6}$

- | | | | | |
|------------------|------------------|------------------|------------------|-----|
| Ⓐ 0 | Ⓐ 1 | Ⓐ 2 | Ⓐ 3 | Ⓐ 4 |
| Ⓑ -1 | Ⓑ -2 | Ⓑ -3 | Ⓑ -4 | |
| Ⓒ $\frac{1}{2}$ | Ⓒ $\frac{3}{2}$ | Ⓒ $\frac{5}{2}$ | Ⓒ $\frac{7}{2}$ | |
| Ⓓ $-\frac{1}{2}$ | Ⓓ $-\frac{3}{2}$ | Ⓓ $-\frac{5}{2}$ | Ⓓ $-\frac{7}{2}$ | |

Note. Let $P(A)$ denote the probability of an event A .

Problem 6 Let A and B be two events, and suppose that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$. Let $P(B|A)$ be the conditional probability of B under A , and suppose that $P(B|A) = \frac{1}{6}$.

- (1) Events A and B are .
- (2) $P(A \cap B) =$ and $P(A \cup B) =$.
- (3) When event C satisfies $P(C) = \frac{1}{2}$ and $A \cap B \subset C$, then $P(A \cap B|C) =$.

Answer Choices

- Ⓐ independent Ⓐ dependent (not independent)
 Ⓑ neither independent nor dependent

Answer Choices ~

- | | | | | | |
|-----------------|-----------------|------------------|------------------|------------------|-------------------|
| Ⓐ 0 | Ⓐ 1 | Ⓐ $\frac{1}{2}$ | Ⓐ $\frac{1}{3}$ | Ⓐ $\frac{2}{3}$ | |
| Ⓑ $\frac{1}{4}$ | Ⓑ $\frac{3}{4}$ | Ⓑ $\frac{1}{5}$ | Ⓑ $\frac{2}{5}$ | Ⓑ $\frac{3}{5}$ | Ⓐ $\frac{4}{5}$ |
| Ⓒ $\frac{1}{6}$ | Ⓒ $\frac{5}{6}$ | Ⓒ $\frac{1}{12}$ | Ⓒ $\frac{5}{12}$ | Ⓒ $\frac{7}{12}$ | Ⓒ $\frac{11}{12}$ |

This page may be used for calculations.

■出題の意図■

応用数学

工学を学ぶために必要な数学の知識及び理解度を測る。

令和8年4月入学(第2回)
山口大学大学院創成科学研究科 博士前期課程(工学系) 入学試験
「応用数学」(Applied Mathematics)

受験区分コード
(Examination code)

受験番号
(Examinee's No.)

氏名
(Name)

※ 解答欄には数値や数式でなく、記号を記入すること(例: ①, i).
(Fill the answer column with the symbol, not the value or formula.)

解答欄 (Answer column)

問1 (Problem 1) 配点: 15点 (Score allocation: 15 points)

1 ①	2 ②	3 ③
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問2 (Problem 2) 配点: 25点 (Score allocation: 25 points)

1 ④	2 ⑤	3 ⑥	4 ⑦	5 ⑧
--------	--------	--------	--------	--------

問3 (Problem 3) 配点: 20点 (Score allocation: 20 points)

1 ④	2 ①	3 ①
--------	--------	--------

問4 (Problem 4) 配点: 20点 (Score allocation: 20 points)

1 ⑤	2 ②	3 ①	4 ②
--------	--------	--------	--------

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← 選択した問題番号を左欄に記入せよ。
(Fill in the left box with the problem number you have chosen.)

問5 (Problem 5) 配点: 20点 (Score allocation: 20 points)

1 ②	2 ①	3 ②	4 ①	5 ⑤	6 ⑤
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問6 (Problem 6) 配点: 20点 (Score allocation: 20 points)

1 ①	2 ④	3 ⑥	4 ⑦
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